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Dynamics of antibaryon-baryon annihilation in the Skyrme model

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Abstract

We examine the dynamics of a baryon number zero lump in the Skyrme picture, as a model for annihilation in the $\bar{N}N$ system. We find that radiation propagates at the causal limit as a localized pulse and that the linearized theory gives a good approximation. Both of these results are contrary to findings in the Sine-Gordon model.

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Figure 1: Time evolution of the amplitudes of the solution of the SG equation (1) with initial values (3).

Well before the advent of QCD, Skyrme [1] proposed a non-linear meson field theory to describe baryons in terms of soliton solutions with nontrivial topologies. He identified the conserved topological charge with the baryon number B and thus built in the conservation law for the baryon number in an elegant way. This model is now called the Skyrme model. In the Skyrme model, a classical solution of the field equation with $B = 1$ is used to describe a nucleon and a solution with $B = 2$ is used for a system of two nucleons. Similarly a solution with $B = 0$ can be used to study a antinucleon-nucleon system. There has been a vast amount of work on $B = 1$ solutions to study static properties of nucleons [2] and on $B = 2$ solutions to study nucleon-nucleon interactions [3]. The Skyrme model has been shown to provide a viable phenomenological description of nucleons and their interactions. In contrast, much less attention has been paid to $B = 0$ solutions. Recently Sommermann *et al* [4] performed a comprehensive calculation for a $B = 0$ solution by integrating the classical equations of motion on a 3D grid numerically. Many interesting features in the time evolution of the antiskyrmion-skyrmion system were observed. For example, the baryon density is found to decay extremely rapidly upon collision of the skyrmion with the anti-skyrmion, close to the causal limit, while the energy distribution remains concentrated in the annihilation region. No satisfactory explanation for the remarkably rapid disappearance of baryon number is offered by Sommermann *et al.* The full calculation of an antiskyrmion-skyrmion collision is complicated and it is difficult to extract simple dynamical insight from it. In this paper we construct a greatly simplified system to explore the dynamics. We find that a spherically symmetric "lump" with $B = 0$ decays quickly, nearly at the causal limit. We also find, surprisingly, that this decay is qualitatively described by the linearized Skyrme equation. By contrast, we find that for the 1+1 Sine-Gordon (SG) equation there is very slow radiative decay of a localized $B = 0$ "lump". The lump sits "ringing" for a long time, slowly radiating its energy away. The linearized SG case radiates its energy more rapidly, but not nearly as quickly as the skyrmion case.

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Figure 2: Time evolution of the amplitudes of the solution of the linear wave equation (2) with initial values (3).

We start with the 1+1 SG model [5]. As a nonlinear field theory itself, the SG model has often been used as a toy model for skyrmions [6]. In fact, soon after the Skyrme model was proposed, Skyrme together with Perring [7] studied time evolution of a two soliton system numerically using the SG model. The equation of motion for the SG model is

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \sin u = 0. \tag{1}$$

In linear approximation, this equation becomes

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u = 0. \tag{2}$$

The topological conserved "baryon number" for the SG model is $B = u(x = \infty) - u(x = -\infty)$. To study the decay of a $B = 0$ system we take the initial configuration

$$u(0, x) = h\frac{aK_1(\sqrt{x^2 + a^2})}{\sqrt{x^2 + a^2}}, \tag{3}$$

with $\frac{\partial u}{\partial t}(0, x) = 0$. This choice allows us to solve the linearized equation (2) analytically. It represents a $B = 0$ lump concentrated around the origin with zero initial velocity. The parameters are chosen such that the lump has twice the energy of a $B = 1$ soliton and thus it represents a $\bar{N}N$ system at threshold. We take h=39 and a=2. In Figures 1 and 2 we show the time evolution of the $B = 0$ solution $u(x)$ for the exact and linear SG equations, respectively. As we can see, the solution to the exact SG equation oscillates very strongly but remains concentrated around the origin and there is very little dissipation. This may be related to the infinite number of conservation laws in the SG model. For the linear solution, there is more radiation. However, some portion of the energy also remains around the origin. Apparently, the linear solution is different from the exact solution. This is not surprising since u is rather large initially and the approximation $\sin u \approx u$ does not

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Figure 3: Time evolution of the amplitude (multiplied by r) of the solution of the Skyrme equation (6) with initial values (9).

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Figure 4: Time evolution of the amplitude (multiplied by r) of the solution of the linear P-wave equation (7) with initial values (9)

apply. Nevertheless both the linear and full equations share the feature that the distribution sits "ringing" near the origin for a long time slowly radiating away its energy.

Naively, we would expect to see similar features in the Skyrme model. This turns out not to be the case. The Lagrangian density of the Skyrme model with zero pion mass is

$$\mathcal{L} = \frac{f_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32 g_\rho^2} Tr[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U][U^\dagger \partial^\mu U, U^\dagger \partial^\nu U], \tag{4}$$

where U is a unitary $SU(2)$ matrix normalized to $U = 1$ at infinity. If we use the hedgehog ansatz for U

$$U = \exp(i\hat{r} \cdot \tau F(r,t)), \tag{5}$$

we obtain the equation of motion

$$(r^2 + 2\sin^2 F)\left(\frac{\partial^2 F}{\partial t^2} - \frac{\partial^2 F}{\partial r^2}\right) = 2r\frac{\partial F}{\partial r} - \sin 2F\left[1 + \left(\frac{\partial F}{\partial t}\right)^2 - \left(\frac{\partial F}{\partial r}\right)^2 + \frac{\sin^2 F}{r^2}\right]. \tag{6}$$

In arriving at the above equation, we used scaled Skyrme units for length and time: $\tilde{r} = f_\pi g_\rho r, \tilde{t} = f_\pi g_\rho t$. For convenience, we still use r and t to denote these scaled quantities. In the linear approximation, this equation becomes

$$\frac{\partial^2 F}{\partial t^2} - \frac{\partial^2 F}{\partial r^2} - \frac{2}{r}\frac{\partial F}{\partial r} + \frac{2}{r^2}F = 0. \tag{7}$$

The boundary values of F are connected to the baryon number B by

$$B = \frac{1}{\pi}(F(0,t) - F(\infty,t)). \tag{8}$$

To model the $\bar{N}N$ system with $B = 0$, we take as an initial configuration

$$F(r,0) = h\frac{r}{(r^2 + a^2)^2}, \tag{9}$$

with $\frac{\partial F}{\partial t}(r,0) = 0$. Again this particular initial condition makes it possible to solve the equation of motion in the linearized model analytically. We choose parameters such that the energy of the lump is twice the energy of a single skyrmion. We take $h = 49.6$ and $a = 2$. Eq. (6) is solved by an implicit difference scheme. The amplitudes $F(r,t)$ (multiplied by r) are shown in Figures 3-4. The most striking feature we see is that there is no radiation in either case. Both solutions propagate as a localized pulse, essentially at the speed of light. Moreover, despite the fact that the initial lump has rather large F and thus invalidates the linear approximation, the linear solution is very close to the exact solution. This implies that in the Skyrme model, decay processes of an $\bar{N}N$ system can be described by a linear meson theory, at least classically. We have performed similar calculations with nonzero pion mass and different initial configurations. We always find very similar solutions for the linear and non-linear equations and find very little radiation. Rather the disturbance propagates away from the origin nearly at the causal limit as seen by Sommermann *et al.* They also find little sensitivity to the pion mass.

That the SG solutions and the Skyrme solutions exhibit very different behaviors should serve as a strong warning that qualitative conclusions one draws from studying the 1+1 SG model do not necessarily apply to the Skyrme model. There are at least two reasons one can give for this difference. First at the level of the linear equation, the difference seen between the skyrmion and SG cases is largely due to the meson mass. In the SG case the mass in dimensionless unit is 1 since it is the only scale in the problem. One can define a massless equation of the linear form, but a massless SG model makes no sense. By contrast in the Skyrme model the mass scale is set by f_π and g_ρ. In those units the pion mass squared is only 0.27. That is why one sees coherent propagation in the linearized Skyrme case and dispersive breakup of the lump in the linearized SG case. The smallness of the pion mass in skyrmion units also accounts for the lack of sensitivity to the addition of this mass term to the Skyrme model that we and Sommermann *et al* find. The dispersive breakup of the SG lump in the full equation is partly due to this mass effect and partly due to the intrinsic nonlinearities of the theory.

These nonlinearities are much less important in the Skyrme case than in the SG case due to the different geometry. It is easy to show that the nonlinear term in the Skyrme case (the difference between (6) and (7)) falls off like $1/r^2$ due to the three dimensional nature of the equation. Hence, no matter how big the nonlinear terms are near the origin, they fall quickly and the linear approximation becomes good. The boundary on the Skyrme equation, Eq. (8), requires that F be zero at the origin, further favoring large r and therefore the domain of the linear equation. It is clear therefore that the rapid evaporation of the energy in a $B = 0$ Skyrme system is a rather general feature of these classical models. The robust nature of the rapid annihilation and its insensitivity to input parameters (recall we are using scaled variables) has also been observed by Sommermann *et al.*

Our results lead to the conclusion that annihilation in the Skyrme model is described by a rapid decay and that the linearized equation describes the evolution of this decay very well. These results may have bearing on the quantum-mechanical annihilation in the antinucleon-nucleon system. First of all it is found experimentally [8] that near threshold the $\bar{N}N$ system couples strongly to purely pionic states. Thus the Skyrme model, which is formulated in terms of pions only, may be a good paradigm for annihilation in this system. The Skyrme picture models QCD in the large number-of-colors (N_C) or mean-field limit. From our results we cannot yet conclude how to describe the formation of pions from the initial $\bar{N}N$ state. However formed, we can conclude that the state should disappear rapidly, nearly at the causal limit. The validity of the linear approximation suggests that after formation we can use a model of non-interacting pions (the quantized from of the linear equation) for the time evolution of the system. The fast decay implies high kinetic energy of the pions, and we thus expect that the total number of pions produced is small compared to 2 GeV$/m_\pi \approx 14$, as born out by experiment [8]. Such a picture in terms of rapid decay into a small number of relativistic, non-interacting pions is very far from the thermodynamic limit.

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